

1 Graph Representations

For the graph above, draw the adjacency list and adjacency matrix representation.

2 DFS and BFS

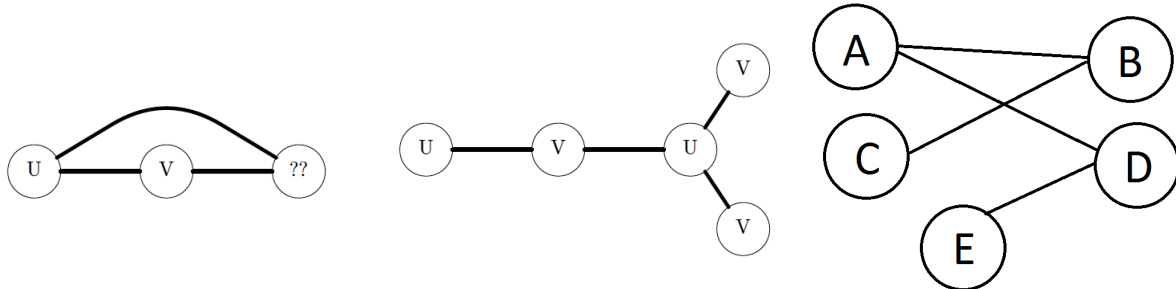
Give the DFS Preorder, DFS Postorder, and BFS order of the graph starting from vertex A. Whenever there is a choice of which node to visit next, visit nodes in alphabetical order.

3 Topological Sorting

Which edge would we need to remove so that there exists a topological sort for the graph above? Give a valid topological sort (Hint: Use DFS Postorder).

4 Graph Algorithm Design: Bipartite Graphs

An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets U and V such that every edge connects an item in U to an item in V . For example, the graphs in the center and on the right are bipartite, whereas the graph on the left is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?



5 Extra Algorithm Design: Shortest Directed Cycles

Provide an algorithm that finds the shortest directed cycle in a graph in $O(EV)$ time and $O(E)$ space, assuming $E > V$.

6 Extra: Daniel's Dare for the Daring

Master programmer, Edwin Edgehands decides to try his hand at implementing the Depth First traversal algorithm. Here is Edgehands' pseudocode:

```
Create a new Stack of Vertices
  Push the start vertex and mark it
  While the fringe is not empty:
    pop a vertex off the fringe and visit it
    for each neighbor of the vertex:
      if neighbor not marked:
        push neighbor onto the fringe
        mark neighbor
```

Your TA, Joshua Shrug claims that the above traversal isn't quite DFS. Give an example graph where it may not traverse in DFS order.